MULTIMEDIA UNIVERSITY

**STUDENT ID NO**

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 1, 2017/2018**

## EMT2046 – ENGINEERING MATHEMATICS IV

(BE, CE, EE, LE, MCE, NE, OPE, RE, TE)

11th OCT 2017

9.00am – 11.00am

(2 Hours)

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**INSTRUCTIONS TO STUDENT**

1. This exam paper consists of **8 pages** (including cover page) with **four questions** and an **appendix** only.
2. Attempt **ALL questions**. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.

## Question 1

(a) A manufacturer produces $x_1$, $x_2$, $x_3$ units of Products A, B and C, respectively, every day. The raw materials required for each product and the profit of each product are tabulated in **Table Q1**. The maximum raw material available every day is 40g, 60g and 50g respectively for raw material P, raw material Q and raw material R. Formulate a linear programming model that can be used to maximize the daily profit subject to the constraints given. (**Do not solve the problem**).

**Table Q1**

| | **Product A** | **Product B** | **Product C** |
|---|---|---|---|
| **Raw material P** | 0g | 1g | 2g |
| **Raw material Q** | 3g | 2g | 2g |
| **Raw material R** | 2g | 0g | 3g |
| **Profit** | RM 1 | RM 3 | RM 2 |

[9 marks]

(b) Consider the following linear programming problem in the standard form.

$$\begin{aligned} &\text{Maximize } z = 2x_1 + x_2 + 3x_3 \\ &\text{subject to:} \quad x_1 + 2x_2 + 2x_3 + s_1 = 20 \\ &\qquad\qquad\quad 2x_1 - x_2 + s_2 = 10 \\ &\qquad\qquad\quad x_1, x_2, x_3, s_1, s_2 \geq 0 \end{aligned}$$

(i) Use simplex method to solve the linear programming problem. [11 marks]

(ii)Convert the linear programming problem to its dual problem. [5 marks]

Continued...

## Question 2

(a) Data packets originating from a computer terminal A are routed to another computer terminal I in four hops. As the network in **Figure Q2-(a)** shows, a packet from A may be relayed to one of two terminals (B or C) in the first hop, one of three terminals (D, E or F) in the second hop, and one of two terminals (G or H) in the third hop. Terminals G and H will then forward the data packet to the receiving terminal I.

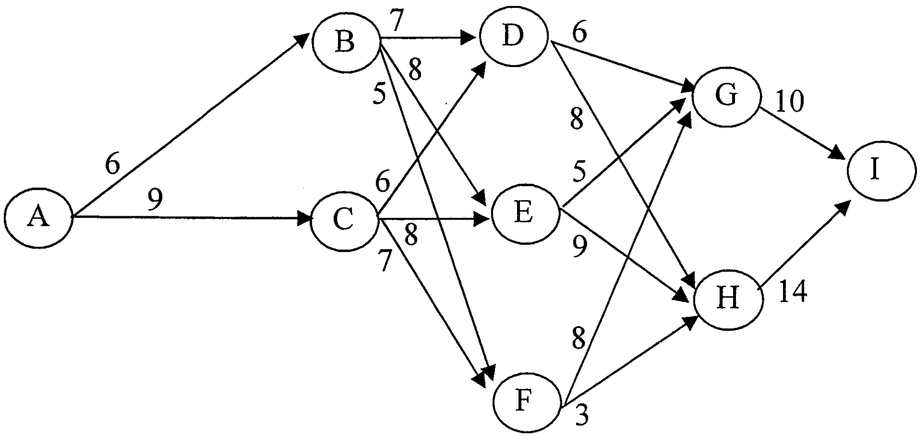


**Figure Q2-(a)**

The number along each link represents the average delay (in milliseconds) experienced by a packet traversing that link. Using dynamic programming, determine a route from A to I that minimizes packet transmission time. What would this minimum transmission time be?

[17 marks]

(b) Apply Kruskal's algorithm to reduce the network in **Figure Q2-(b)** below to a minimum spanning tree. Draw the final network with its surviving edges.

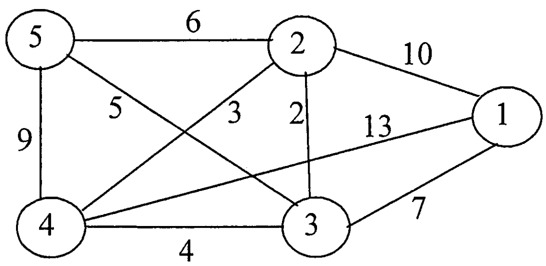


**Figure Q2-(b)**

[8 marks]

**Continued...**

**Question 3**

(a) Use composite Simpson's rule to approximate $\int_{0}^{0.9}(2+\cos 2x)\,dx$ with step size $h = 0.15$ and calculate the absolute error. Round your answer to four decimal places.

[13 marks]

(b) Approximate a positive root of $x^2 - 2x - 2 = 0$ by using Newton-Raphson's method with initial value of 3. Use the termination criterion $\left|\frac{x_n - x_{n-1}}{x_n}\right| \le \varepsilon$ with $\varepsilon = 0.0005$.

[12 marks]

**Continued...**

## Question 4

A vending machine has just been set up in a supermarket. As part of its maintenance program, a diagnostic is carried out every month to assess whether the machine is operating satisfactorily. Its status will then be graded as A (best), B, C or D (worst). The machine will be sent for repair when the status deteriorates to grade D. Assume that the monthly status of the machine may be modeled by a Markov chain with state space S = {A, B, C, D}, and that its transition probability matrix is given by

$$\boldsymbol{P} = \begin{bmatrix} u & v & 0.2 & 0.1 \\ 0 & 0.5 & 0.3 & 0.2 \\ 0 & 0 & 2v & u \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

where $u$ and $v$ are constants.

(a) Find the values of $u$ and $v$.

[3 marks]

(b) Draw the state transition diagram.

[4 marks]

(c) Decompose the state space into equivalence classes. Determine whether each class is recurrent or transient

[4 marks]

(d) Suppose the vending machine initially operates at grade A. Find the probability that, after 2 months, it will be
(i) operating at grade B.

[3 marks]

(ii)sent for its first repair.

[3 marks]

(e) The manufacturer of the vending machine wishes to predict its operational status in the long run.
(i) Write the system of equations that yields the long run probabilities (i.e., $\pi_A$ , $\pi_B$ , $\pi_C$ and $\pi_D$ ).

[5 marks]

(ii)After long run, the probability that the machine is sent for repair is 0.19. Use this information to solve the system in part (e)-(i). (Round your answer to 2 decimal places)

[3 marks]

**Continued...**

**APPENDIX**

**TABLE OF FORMULAS**

1. The *n*th Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0) \cdots (x - x_{k-1})$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0)$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n)$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

**Continued...**

7. Central difference formula

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

with the error term

$$\left|\frac{h^2}{6} f^{(3)}(c_x)\right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{h}{2}(f(a)+f(b)) - \frac{h^3 f''(\xi)}{12}$$

for some $\xi$ in $(a, b)$ and $h = b - a$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)\right]$$

for some $\xi$ in $(a, b)$ and $h = \dfrac{b-a}{n}$, with the error term is $\left|\dfrac{(b-a)h^2 f''(\xi)}{12}\right|$

10. Simpson's rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{h^5}{90} f^{(iv)}(\xi)$$

for some $\xi$ in $(a, b)$ and $h = \dfrac{b-a}{2}$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j})+4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1})+f(b)\right]$$

for some $\xi$ in $(a, b)$ and $h = \dfrac{b-a}{n}$, with the error term $\left|\dfrac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$

12. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0,1,2,\ldots$$

13. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\dfrac{h^2}{2} Y''(\xi_i)$ for some $\xi_i$ in $(x_i, x_{i+1})$.

**Continued ...**

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

15. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1),$$
$$k_3 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

**End of Paper**